EE3124 Test 2 with Solutions

Name: **Student No.:**

Q1 – – What is armature reaction?

– Compare the motor windings in Synchronous Motors?

– What is the relationship between electrical frequency and mechanical rotation speed for a synchronous ac machine? (15 pts)

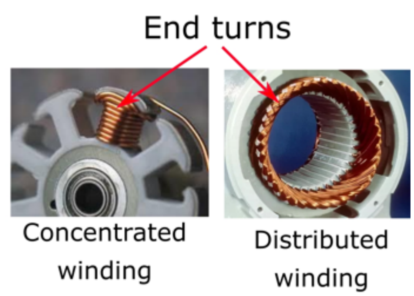
Solution

1. Armature reaction: The distortion of the air-gap magnetic field by the current flowing in the stator.

Motor windings in Synchronous Motors

Key Differences

Feature	Concentrated Winding	Distributed Winding
Structure	Wound around a single tooth	Spread over multiple slots
Back-EMF	Trapezoidal	Sinusoidal
Torque Ripple	Higher	Lower
End Windings	Short	Long
Manufacturing	Simple	Complex



Concentrated winding:
- **Advantages:** Shorter end-windings (lower copper usage), higher efficiency in specific designs, and easier to manufacture.
- **Applications:** Suitable for motors with shorter axial length and large diameter, such as servo motors.
- **Disadvantages:** Higher harmonic content, higher torque ripple, and, in some cases, lower efficiency.

Distributed winding:
- **Advantages:** Produces a near-sinusoidal back-EMF, lower harmonic content, reduced torque ripple, and better thermal management.
- **Applications:** Ideal for high-speed, high-efficiency, and low-vibration applications.
- **Disadvantages:** Longer end-windings (larger copper usage) and more complex manufacturing

2.

3. It is possible to relate the electrical frequency in hertz to the resulting mechanical speed of the magnetic fields in revolutions per minute (rpm). This relationship is

$$f_e = \frac{P}{2} f_m$$

$$n_m(rpm) = \frac{2 \times f_e}{P} \times 60$$

Q2 – A 6-kV 3-phase Y-connected turbo-synchronous generator, of synchronous reactance of 0.6 Ω/phase, is supplying 30 MVA at 0.9 p.f. lagging to a large power system.

(a) Calculate the phase voltage and current of the generator, plot phasor diagram. (15 pts)

(b) If the steam supply is cutoff, explain what takes place, and determine the current the generator will then carry assuming negligible losses? Plot phasor diagram. (15 pts)

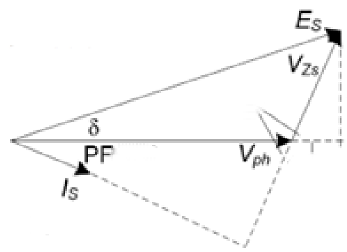
Solution

Since the generator is star-connected, phase currents equal line currents.

$$I_s = \frac{30\times10^6 \angle \cos^{-1} 0.9}{\sqrt{3}\times 6000}$$
$$= 2886.75\angle -25.84^\circ\ A$$

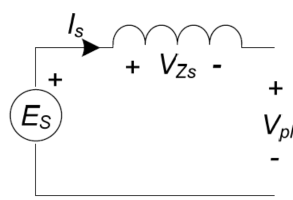
But phase voltages (V_{ph})equal to $\frac{V_{line}}{\sqrt{3}}$.The E_A is

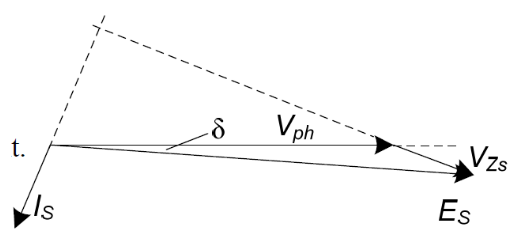
$$E_s = \frac{6000}{\sqrt{3}} + \left(0.6\angle 90^\circ\right)\left(2886.75\angle -25.84^\circ\right)$$
$$= 4219.03 + j1558.87$$
$$= 4497.81\angle 20.28$$
$$\left|E_s\right| = 4497.81\ V$$



As the steam is cutoff, the input power reduces to zero, i.e. $\sin\delta >> 0$. Therefore, the rotor slows down and E_S falls in-phase (behind) the phase voltage V_{ph}. A lagging torque angle induced torque equals the load torque on its shaft. the generator is now operating as a motor. Assuming nebligible losses, the current carry in

$$I_s = \frac{E_S - V_{ph}}{X_S} = \frac{4497.81 - \frac{6000}{\sqrt{3}}}{0.6} = 1722.85\text{ A}$$





Q3 – A 380-V, 60 Hz, two pole pair synchronous motor draws 30 A from the line at unity power factor and full load. Assuming that the motor is lossless, answer the following questions:

(a) What is the output torque of this motor? Express the answer in newton-meters. (10 pts)

(b) What must be done to change the power factor to 0.8 leading? Explain your answer, using phasor diagrams. (5 pts)

(c) What will the magnitude of the line current be if the power factor is adjusted to 0.8 leading? (5 pts)

Solution

(a) If this motor is assumed lossless, then the input power is equal to the output power. The input power to this motor is

$$P_{\text{IN}} = \sqrt{3}V_T I_L \cos\theta = \sqrt{3}(380\text{V})(30\text{A})(1.0) = 19.75 \text{ kW}$$

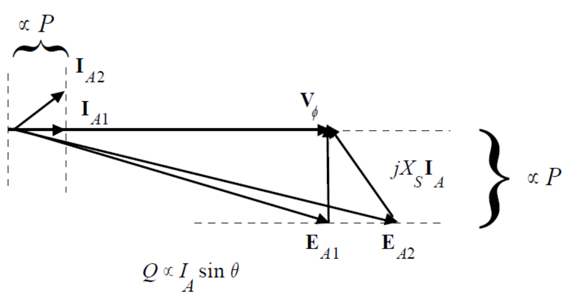
The rotational speed of the motor is

$$n_m = \frac{120 f_{se}}{P} = \frac{120(60\text{Hz})}{4} = 1800 \text{ rpm}$$

The output torque would be

$$\tau_{\text{LOAD}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{19.75 \text{ kW}}{(1800 \text{ rpm})\left(\frac{1 \text{ min}}{60\text{s}}\right)\left(\frac{2\pi \text{ rad}}{1\text{r}}\right)} = 104.78 \text{ Nm}$$

(b) To change the motor's power factor to 0.8 leading, its field current must be increased. Since the power supplied to the load is independent of the field current level, an increase in field current increases E_A while keeping the distance $E_A\sin\delta$ constant. This increase in E_A changes the angle of the current IA, eventually causing it to reach a power factor of 0.8 leading.



(c) The magnitude of the line current will be

$$I_L = \frac{P}{\sqrt{3}V_T \text{PF}} = \frac{19.75 \text{ kW}}{\sqrt{3}(380\text{V})0.8} = 37.51\text{A}$$

Q4 - A 380-V four-pole 50-Hz Y-connected wound-rotor induction motor is rated at 80 hp. Its equivalent circuit components are

$R_1 = 0.100$ $R_2 = 0.070$ $X_M = 10.0$

$X_1 = 0.210$ $X_2 = 0.210$

$P_{mech} = 1500$ W $P_{core} = 1000$ W $P_{misc} \approx 0$

For a slip of 0.1, find

(a) The line current I_L (5 pts)

(b) The stator copper losses P_{SCL} (5 pts)

(c) The air-gap power P_{AG} (5 pts)

(d) The power converted from electrical to mechanical form P_{conv} (5 pts)

(e) The induced torque τ_{ind} (5 pts)

(f) The load torque τ_{load} (5 pts)

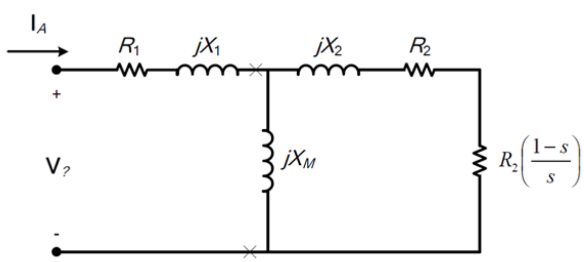
(g) The overall machine efficiency (5 pts)

(h) The motor speed in revolutions per minute and radians per second. (5 pts)

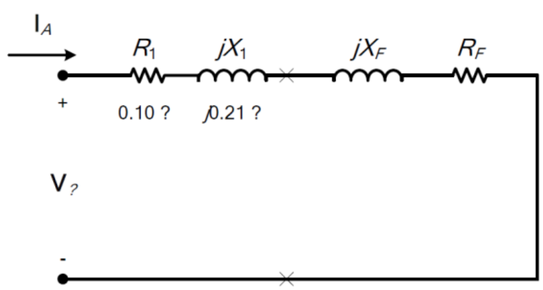
(Any correct 7 answers will have full score) (35 pts)

Solution

The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j10\Omega} + \frac{1}{0.7 + j0.21}} = 0.6684 + j0.2515 = 0.714\angle 20.62$$

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{220\angle 0^\circ \text{ V}}{0.10\Omega + j0.21\Omega + 0.6684\Omega + j0.2515\Omega}$$

$$I_L = I_A = 245.44\angle -30.99^\circ \text{ A}$$

(b) The stator copper losses are

$$P_{\text{sCL}} = 3I_A^2 R_1 = 3(245.44A)^2(0.10 \quad \Omega) = 18.072kW$$

(c) The air-gap P_{AG}

$$P_{AG} = P_{in} - P_{SCL} - P_{core} = \sqrt{3} \times V_L \times I_L \times \cos(-30.99) - 18072 - 1000 = 119.412 \text{ kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{\text{AG}} = (1-0.1)119.412 \text{ kW} = 107.471 \text{ kW}$$

(e) The induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{119.412 \text{ kW}}{(1500\text{r}/\text{min})\left(\frac{2\pi\text{rad}}{1\text{r}}\right)\left(\frac{1\text{min}}{60\text{s}}\right)} = 760.2 \text{ Nm}$$

(f) The output power of this motor is

$P_{OUT}= P_{conv}- P_{mech}- P_{misc}=$ 107.471 –1.5 – 0 kW= 105.971 kW

The output speed is

$$n_m = (1-s)n_{\text{sync}} = (1-0.1)(1500\text{r}/\text{min}) = 1350 \text{ r}/\text{min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{105.971\text{kW}}{(1350\text{r}/\text{min})\left(\frac{2\pi\text{rad}}{1\text{r}}\right)\left(\frac{1\text{min}}{60\text{s}}\right)} = 749.6 \text{ Nm}$$

(g) The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{P_{\text{OUT}}}{3V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{105.97 \text{ kW}}{3(220\text{V})(245.44\text{A})\cos -30.99^\circ} \times 100\% = 76.31\%$$

(h) The motor speed in radians per second is

$$\omega_m = (1350\text{r}/\text{min})\left(\frac{2\pi\text{rad}}{1\text{r}}\right)\left(\frac{1\text{min}}{60\text{s}}\right) = 141.37 \text{ rad}/\text{s}$$